INCONTRO I - II

① $|x(x-3)| > x^2-1$

$\Updownarrow$

$x(x-3) > x^2-1 \quad \vee \quad x(x-3) < -x^2+1$

SI SEGUE LO SCHEMA:

$|A(x)| > B(x) \Longleftrightarrow A(x) > B(x) \ \vee \ A(x) < -B(x)$

$|A(x)| < B(x) \quad \begin{cases} A(x) < B(x) \\ A(x) > -B(x) \end{cases}$

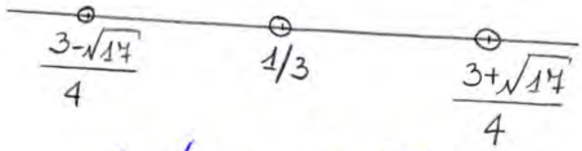
$x^2-3x > x^2-1 \quad \vee \quad x^2-3x < -x^2+1$

$3x < 1 \quad \vee \quad 2x^2-3x-1 < 0$

$\Delta = 9 - 4\cdot 2\cdot(-1)$

$x_{1/2} = \dfrac{3 \pm \sqrt{17}}{4}$

$x < \dfrac{1}{3} \quad \vee \quad \dfrac{3-\sqrt{17}}{4} < x < \dfrac{3+\sqrt{17}}{4}$



$S = \left(-\infty, \dfrac{3+\sqrt{17}}{4}\right)$

② $|(x+1)|x-1| - x| \leq 2$

$\begin{cases} x-1 \geq 0 \\ |x^2-1-x| \leq 2 \end{cases} \vee \begin{cases} x-1 < 0 \\ |1-x^2-x| \leq 2 \end{cases}$

$\begin{cases} x \geq 1 \\ x^2-1-x \leq 2 \\ x^2-1-x \geq -2 \end{cases} \vee \begin{cases} x < 1 \\ x^2+x-1 \geq -2 \\ x^2+x-1 \leq 2 \end{cases}$

ⓐ ⓑ

ⓐ $\begin{cases} x \geq 1 \\ x^2-x-3 \leq 0 \\ x^2-x+1 \geq 0 \end{cases}$ → $x_1 = \dfrac{1-\sqrt{13}}{2}$ → $x_2 = (1+\sqrt{13})/2$

$\forall x \in \mathbb{R}$ → parabola associata

$\Delta = 1 - 4\cdot 1 < 0$

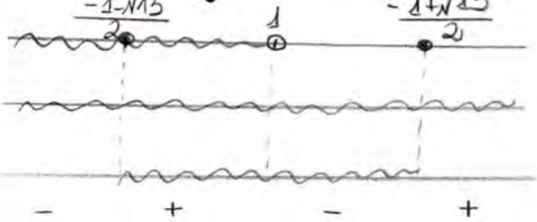
La parabola non tocca l'asse x

ⓐ $1 \leq x \leq \dfrac{1+\sqrt{13}}{2}$

ⓑ $\begin{cases} x < 1 \\ x^2+x+1 \geq 0 \quad \forall x \in \mathbb{R} \\ x^2+x-3 \leq 0 \end{cases}$

$\Delta = 1 - 4\cdot 1(-3) = 13$

$x_{1/2} = \dfrac{-1 \pm \sqrt{13}}{2}$



ⓑ $-\dfrac{1-\sqrt{13}}{2} \leq x < 1$

$S = S_1 \vee S_2 = 1 \leq x \leq \dfrac{1+\sqrt{13}}{2} \vee \dfrac{-1-\sqrt{13}}{2} \leq x < 1$

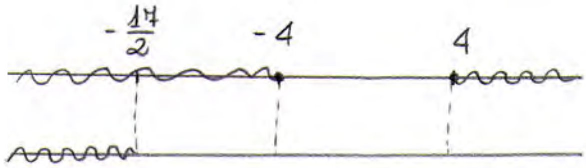
$= \left[-\dfrac{1+\sqrt{13}}{2}; \dfrac{1+\sqrt{13}}{2}\right]$

③ $\sqrt{x^2-16} > |x+1|$

$$\begin{cases} x^2-16 \geq 0 \\ x^2-16 > (|x+1|)^2 \end{cases}$$ si può togliere il valore assoluto perchè $(|a|)^2 = a^2$

$$\begin{cases} x \leq -4 \vee x \geq 4 \\ x^2-16 > \cancel{x^2}+2x+1 \end{cases}$$

$$\begin{cases} x \leq -4 \vee x \geq 4 \\ 2x < -17 \end{cases}$$



$S = \left(-\infty ; \frac{17}{2}\right)$

④ $|2x-4| < 1$

$|2(x-2)| < 1$

$|2| \cdot |x-2| < 1$

$2 \cdot |x-2| < 1$

$|x-2| < 1/2$

→ Metodo algebrico:

$-\frac{1}{2} < x-2 < \frac{1}{2}$

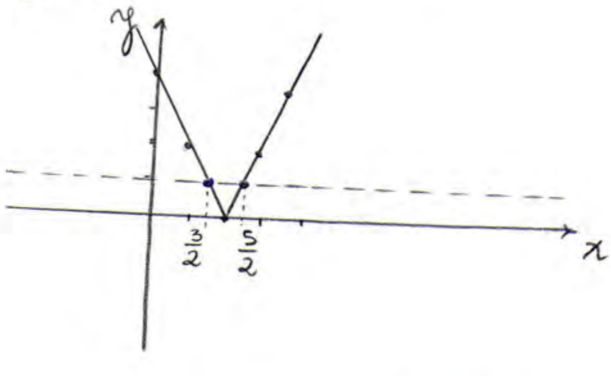
$$\begin{cases} x-2 > -\frac{1}{2} \\ x-2 < \frac{1}{2} \end{cases} \qquad \begin{cases} x > 2-\frac{1}{2} \\ x < 2+\frac{1}{2} \end{cases} \qquad \begin{cases} x > \frac{3}{2} \\ x < \frac{5}{2} \end{cases} \qquad \frac{3}{2} < x < \frac{5}{2}$$

→ Metodo analitico:

$y = |2x-4|$

$y = 1$

x	y_1	y_2
0	-4	4
1	-2	2
2	0	0
3	2	-2

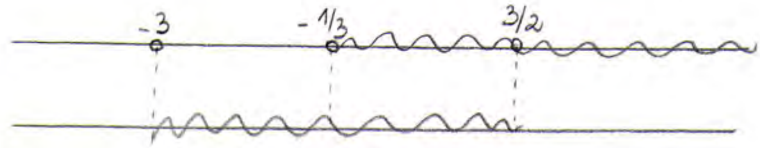


⑤ $|x^2+3x-4| < 5-x^2$

$x^2-5 < x^2+3x-4 < 5-x^2$

$$\begin{cases} x^2+3x-4 > x^2-5 \\ x^2+3x-4 < 5-x^2 \end{cases} \qquad \begin{cases} 3x > -1 \\ 2x^2+3x-9 < 0 \end{cases} \qquad \begin{cases} x > -1/3 \\ -3 < x < \frac{3}{2} \end{cases} \qquad -\frac{1}{3} < x < \frac{3}{2}$$

$x_{1|2} = \frac{-3 \pm \sqrt{81}}{4}$ → $x_1 = \frac{-3-9}{4} = -3$ → $x_2 = \frac{-3+9}{4} = \frac{6}{4} = \frac{3}{2}$



$S = \left(-\frac{1}{3} ; \frac{3}{2}\right)$

⑥ $\left|\dfrac{\cos 2x}{\operatorname{sen}x}\right| < 1$

RICORDA:

$\cos 2x = \cos^2 x - \operatorname{sen}^2 x$
$= 1 - 2\operatorname{sen}^2 x = 2\cos^2 x - 1$

$$-1 < \frac{\cos 2x}{\operatorname{sen}x} < 1$$

$$\begin{cases} \dfrac{1-2\operatorname{sen}^2 x}{\operatorname{sen}x} > -1 \\[2ex] \dfrac{1-2\operatorname{sen}^2 x}{\operatorname{sen}x} < 1 \end{cases} \qquad \begin{cases} \dfrac{1-2\operatorname{sen}^2 x+\operatorname{sen}x}{\operatorname{sen}x} > 0 \\[2ex] \dfrac{1-2\operatorname{sen}^2 x-\operatorname{sen}x}{\operatorname{sen}x} < 0 \end{cases}$$

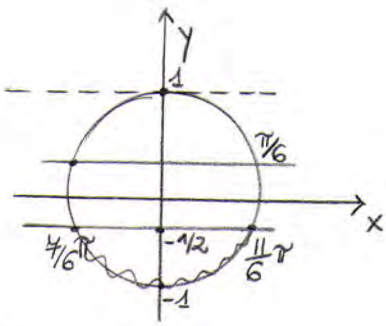
$$\begin{cases} \dfrac{2\operatorname{sen}^2 x-\operatorname{sen}x-1}{\operatorname{sen}x} < 0 \quad ① \\[2ex] \dfrac{2\operatorname{sen}^2 x+\operatorname{sen}x-1}{\operatorname{sen}x} > 0 \quad ② \end{cases}$$

① $2\operatorname{sen}^2 x - \operatorname{sen}x - 1 > 0$ $\qquad \operatorname{sen}x > 0$

$2t^2 - t - 1 > 0$

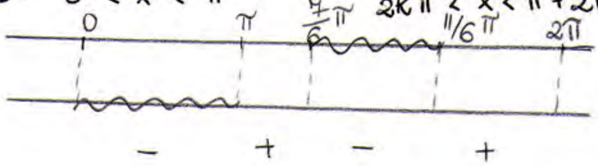
$t_{1/2} \to -\frac{1}{2}$, $\downarrow 1$ $\Rightarrow$ $t < -\frac{1}{2} \vee t > 1$

$\operatorname{sen}x < -\frac{1}{2}$



N: $\frac{7}{6}\pi < x < \frac{11}{6}\pi \to \frac{7}{6}\pi + 2k\pi < x < \frac{11}{6}\pi + 2k\pi$ $\qquad \wedge\ x = \frac{\pi}{2}$

D: $0 < x < \pi \to 2k\pi < x < \pi + 2k\pi$

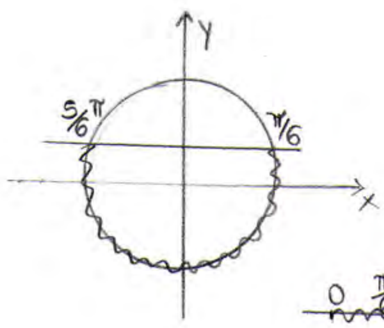


$0 \neq 2k\pi < x < \pi + 2k\pi \ \vee\ \frac{7}{6}\pi + 2k\pi < x < \frac{11}{6}\pi + 2k\pi$

② $2t^2 + t - 1 > 0$ $\qquad \operatorname{sen}x > 0$

$t_{1/2} \to \frac{1}{2}$, $\searrow 1$

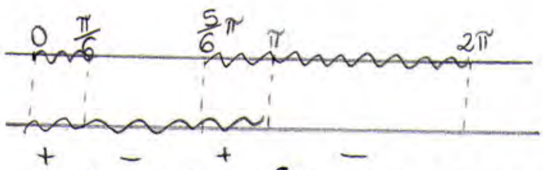
$t < \frac{1}{2} \vee t > 1 \to \operatorname{sen}x < \frac{1}{2}$



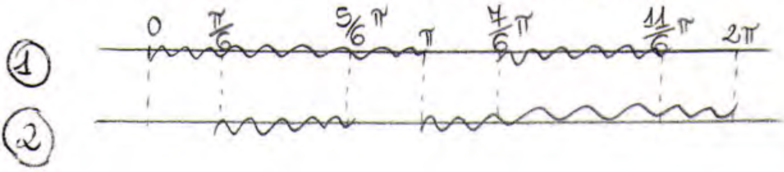
N: $0 < x < \frac{\pi}{6} \ \vee\ \frac{5}{6}\pi < x < 2\pi \to$

$2k\pi < x < \frac{\pi}{6} + 2k\pi \ \vee\ \frac{5}{6}\pi + 2k\pi < x < 2\pi + 2k\pi$ $\quad \wedge\ x = \frac{3}{2}\pi$

D: $0 < x < \pi \to 2k\pi < x < \pi + 2k\pi$



$\frac{\pi}{6} + 2k\pi < x < \frac{5}{6}\pi + 2k\pi \ \vee\ \pi + 2k\pi < x < 2\pi + 2k\pi$



$\mathcal{S} = S_1 \cap S_2$

$\frac{\pi}{6} + 2k\pi < x < \frac{5}{6}\pi + 2k\pi \ \vee\ \frac{7}{6}\pi + 2k\pi < x < \frac{11}{6}\pi + 2k\pi$

$\wedge\ x = \pi/2;\ x = 3/2\pi$

⑦ $A \subseteq \mathbb{R}$, $a = \sup A \iff$ 1) $a \geq x$, $\forall x \in A$
2) $\forall \varepsilon > 0$, $\exists\, \bar{x} \in A : a - \varepsilon < \bar{x}$

$b = \inf A \iff$ 1) $b \leq x \quad \forall x \in A$
2) $\forall \varepsilon > 0 \quad \exists\, \bar{x} \in A : \bar{x} < b + \varepsilon$

$$A = \left\{ x_n = \frac{n^2}{n+3} \quad , n \in \mathbb{N} \right\}$$

$\rightarrow n = 0, \quad x_n = 0$
$\rightarrow n > 0, \quad \dfrac{n^2}{n+3} > 0$

Il num. è molto più grande del denom. quindi la funzione aumenta all'aumentare di n. $\rightarrow$ Posso supporre: $\sup A = +\infty$
$\Rightarrow$ Devo dim. che non riesco a limitare a destra l'insieme.

$\forall M > 0 \qquad \dfrac{n^2}{n+3} > M$

$n^2 - nM - 3M > 0 \qquad \Delta = M^2 + 12M$ (num. pos.)

$$n_{1/2} = \frac{M \pm \sqrt{M^2 + 12M}}{2}$$

$$n < \frac{M - \sqrt{M^2+12M}}{2} \quad \vee \quad n > \frac{M + \sqrt{M^2+12M}}{2}$$

⑧ $A = \left\{ (-1)^n \dfrac{n+1}{n}, n \in \mathbb{N} \right\}$, $x_n = (-1)^n \cdot \dfrac{n+1}{n}$

n pari e $n \geq 2$ (n non può essere 0) $\rightarrow x_n = \dfrac{n+1}{n} \rightarrow$ daranno il massimo

n dispari e $n \geq 1$ $\rightarrow x_n = -\dfrac{n+1}{n} \rightarrow$ " " minimo

- Se $n = 2$, $x_n = \dfrac{3}{2}$

$n = 3$, $x_n = \dfrac{4}{3}$ ipotesi: $1 < x_n < \dfrac{3}{2}$

$\inf A = -2$
$\sup A = \dfrac{3}{2}$

$\dfrac{n+1}{n} \leq \dfrac{3}{2} \qquad (n+1) \cdot 2 < 3n$

$2n + 2 < 3n$

$-n < -2 \rightarrow n \geq 2 \quad \Rightarrow x_n = \dfrac{3}{2} = MAX$

- Se $n = 1$, $x_n = -2$ ipotesi: $-2 < x_n < -1$

$-\dfrac{n+1}{n} \geq -2 \qquad -n-1 \geq -2n$

$n \geq 1$ se è il min. di tutti gli el. neg. sarà < anche di quelli pos.

9) $A = \left\{ \frac{2n^2-1}{n^2} , n \in \mathbb{N} \wedge n \neq 0 \right\}$

$$\frac{2n^2-1}{n^2} = \frac{2\cancel{n^2}}{\cancel{n^2}} - \frac{1}{n^2} = 2 - \frac{1}{n^2}$$

$n \to \infty \quad , \quad A = 2 \quad \to \quad \boxed{\sup A = 2}$

$n = 1 \quad , \quad A = 1$

$n = 2 \quad , \quad A = 2 - \frac{1}{4} = \frac{7}{4}$ se $n \uparrow$, mi avvicino a 2

$\Rightarrow \boxed{\inf A = 1}$ Si può dim. Es.: $2 - \frac{1}{n^2} > 2 - \varepsilon \to \frac{1}{n^2} < \varepsilon \quad n^2 > \frac{1}{\varepsilon}$

10) $A = \{ n(n-2)(n-5), n \in \mathbb{N} \wedge n \neq 0 \}$

$n = 1 \to 1 \cdot (1-2)(1-5) = 4$

$2 < n < 5 \to$ neg.

$n > 5 \to$ pos.

$n \to \infty \quad , \quad A \to \infty$ $\quad \sup A = +\infty$

$n = 2 \to A = 2 \cdot 0 \cdot (2-5) = 0$

$n = 3 \to A = 3 \cdot 1 \cdot (-2) = -6$

$n = 4 \to A = 4 \cdot 2 \cdot (-1) = -8$ $\quad \inf A = -8$

$n = 5 \to A = 0$

INCONTRO III - IV

① Dire in quali punti le funzioni sono continue.

ⓐ $f(x) = \dfrac{x^2-2}{x^2+1}$

x^2-2, x^2+1 sono funzioni continue su tutto $\mathbb{R}$ (somma e prodotti di funz. continue)

$\Rightarrow f(x)$ continua $\forall x \in \mathbb{R} \wedge x^2+1 \neq 0$. N.B.: $x^2+1>0$

$\Rightarrow \mathcal{D} = \mathbb{R}$

ⓑ $f(x) = \dfrac{e^{x \cdot \operatorname{sen}x - \cos x}}{1+\cos x + e^{-x}}$

$x \cdot \operatorname{sen}x - \cos x$: continua su tutto $\mathbb{R}$ perché somma di f.ni continue

$e^{x \cdot \operatorname{sen}x - \cos x}$: " " " " " " composiz. " " "

$e^{-x} > 0$,
$1+\cos x \geq 0$ $\forall x \in \mathbb{R}$ $\Big\} \rightarrow$ il denominatore non è mai nullo

$\Rightarrow \mathcal{D} = \mathbb{R}$

ⓒ $f(x) = \dfrac{e^{2x}+x^2 \operatorname{sen}x - 2}{(e^{\operatorname{sen}x}-\operatorname{sen}x)\cdot\cos x}$

$e^{2x}+x^2 \cdot \operatorname{sen}x - 2$
$(e^{\operatorname{sen}x}-\operatorname{sen}x)\cdot\cos x$ f. continue su tutto $\mathbb{R}$ (Σ, prodotto e composiz. di f.ni continue)

$(e^{\operatorname{sen}x}-\operatorname{sen}x)\cdot\cos x \neq 0$ " " " " " " " "

$e^t \geq 1+t > t \rightarrow$ se $t = \operatorname{sen}x \rightarrow e^{\operatorname{sen}x}-\operatorname{sen}x > 0 \ \forall x \in \mathbb{R}$

$\cos x = 0 \longrightarrow x = \frac{\pi}{2}+2k\pi$
$\searrow x = \frac{3}{2}\pi + 2k\pi$

$\Rightarrow \mathcal{D} = \mathbb{R} \setminus \{x : x = \frac{\pi}{2}+k\pi,\ k \in \mathbb{Z}\}$

② $A = \left\{ \arctan\left(\frac{4-4n}{n^2}\right) : n \in N^+ \right\}$

Calcolare gli estremi sup. e inf.

Ip.

$B = \frac{4-4n}{n^2} \rightarrow B \subset [-1, 0]$

$\frac{4\cancel{n}\left(\frac{1}{n}-1\right)}{n^2}$

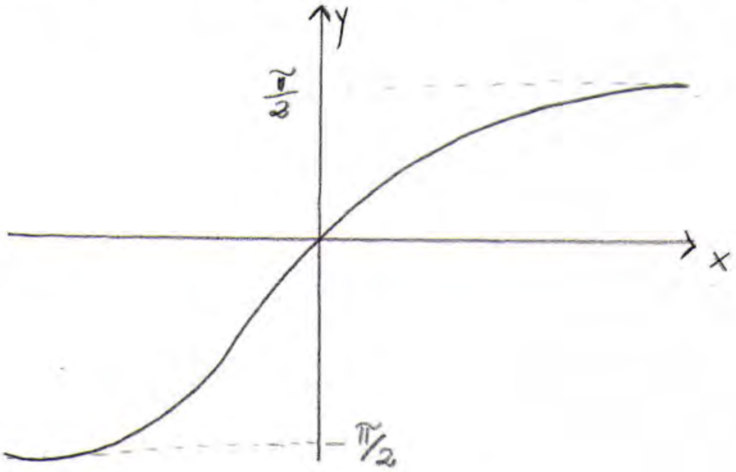
$n \to \infty \quad \frac{4}{\infty} \to 0$

$n = 1 \to 0$

$n = 2 \to -1$

$n = 3 \to -\frac{8}{9} \quad$ se $n \uparrow \to B \to 0$

$\Rightarrow$ A ha max e min. dati da 0 e -1.



$y = \arctan x$ (CRESCENTE IN $(0, \pi)$)

$\inf \left\{ \arctan\left(\frac{4-4n}{n^2}\right) : n \in \mathbb{N}^+ \right\} = \min \left\{ \arctan\left(\frac{4-4n}{n^2}\right) : n \in \mathbb{N}^+ \right\} = \arctan(-1) = -\frac{\pi}{4}$

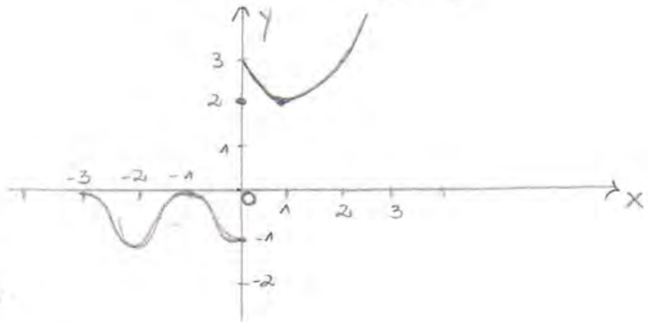
$\sup \left\{ \arctan\left(\frac{4-4n}{n^2}\right) : n \in \mathbb{N}^+ \right\} = \max \left\{ \arctan\left(\frac{4-4n}{n^2}\right) : n \in \mathbb{N}^+ \right\} = \arctan(0) = 0$

③ . Studiare i lim. della funzione

$$f(x) = \begin{cases} x^2 - 2x + 3 & \text{se } x > 0 \\ 2 & \text{se } x = 0 \\ \operatorname{sen} x - 1 & \text{se } x < 0 \end{cases}$$

PARABOLA

$V = \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$



per $x \to 0^+$, $x \to 0^-$, $x \to 0$.

. Studiare il Dominio

$\to (x^2 - 2x + 3$ e $\operatorname{sen} x - 1)$: funzioni continue

quindi lim $x \to 0^+$ e $x \to 0^-$ esistono.

$\lim_{x \to 0^+} f(x) = x^2 - 2x + 3 = 3$

$\lim_{x \to 0^-} f(x) = \operatorname{sen} x - 1 = -1$

lim dx $\neq$ lim sin quindi $\lim_{x \to 0}$ non $\exists$

$\to \mathbb{D} = \mathbb{R} \setminus \{0\}$

$\hookrightarrow$ unico punto di non continuità

④ Dire per quali valori il parametro a la funzione risulta continua per $x=0$

$$f(x)=\begin{cases} x^2-2x+3 & \text{se } x>0 \\ a(\operatorname{sen}x-1) & \text{se } x\leq 0 \end{cases}$$

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}(x^2-2x+3)=3$$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-} a\cdot(\operatorname{sen}x-1)=-a$$

Affinché f risulti continua in $x=0$, lim. sin = lim dx

$\Rightarrow -a=3 \rightarrow a=-3$

⑤ Calcolare se esistono i seguenti limiti

ⓐ $\lim_{x\to +\infty} e^x \operatorname{sen}x$

per $x\to+\infty$, $e^x\to+\infty$
per $x\to+\infty$, $\operatorname{sen}x$ non ha limite
IL LIMITE NON ESISTE

ⓑ $\lim_{x\to 0} e^x\cdot \operatorname{sen}x$

per $x\to 0$, $e^x\to 1$
per $x\to 0$, $\operatorname{sen}x\to 0$
Per il teorema del prodotto del limite:
$\ell=\ell_1\cdot\ell_2=0\cdot 1=0$

ⓒ $\lim_{x\to -\infty} e^x\cdot\operatorname{sen}x$

per $x\to-\infty$, $e^x\to 0$
per $x\to-\infty$, $\operatorname{sen}x$ non ha limite

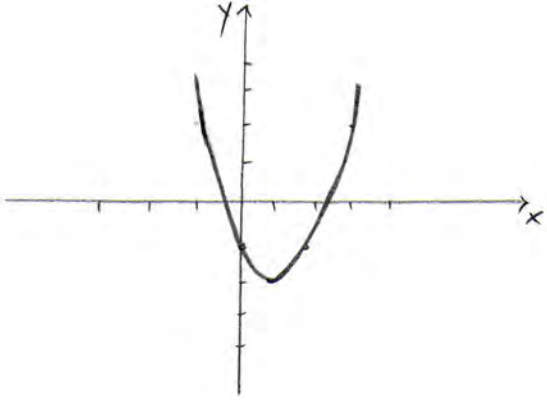
N.B.: $\operatorname{sen}x$ è limitata, il lim. di un infinitesimo con una funzione limitata esiste ed è $=$ a 0.
Ovvero, $|\operatorname{sen}x|\leq 1$, quindi:
$0\leq|e^x\operatorname{sen}x|=|e^x|\cdot|\operatorname{sen}x|\leq e^x\to 0$ per $x\to 0$

Per il teorema del confronto:
$|e^x\cdot\operatorname{sen}|\to 0$ e $e^x\operatorname{sen}x\to 0$

ⓓ $\lim_{x\to -\infty}(e^x-1)\cdot\cos x$

per $x\to-\infty$, $e^x\to 0$, $e^x-1\to -1$
per $x\to-\infty$, $\cos x$ non ha limite
IL LIMITE NON ESISTE

⑥ $\lim_{x \to -1} \quad f(x) = x^2 - 2x - 1$



$$\lim_{x \to -1} (-1)^2 - 2(-1) - 1 =$$
$$1 + 2 - 1 = 2$$

⑦ ⓐ $\lim_{x \to +\infty} \dfrac{x - 2x^2}{3x^2 - 2x}$

$$\lim_{x \to +\infty} \frac{\cancel{x^2}\left(\frac{1}{x} - 2\right)}{\cancel{x^2}\left(3 - \frac{2}{x}\right)}$$

Utilizzando il teorema sul limite del quoziente:

$$\lim_{x \to +\infty} = -\frac{2}{3}$$

ⓑ $\lim_{x \to 0} \dfrac{x - 2x^2}{3x^2 - 2x} \qquad \lim_{x \to 0}$ quindi raccolgo x

$$\lim_{x \to 0} \frac{\cancel{x}(1 - 2x)}{\cancel{x}(3x - 2)} = -\frac{1}{2}$$

ⓒ $\lim_{x \to -\infty} \dfrac{2x - 3x^2}{x^3 + x^2}$

$$\lim_{x \to -\infty} \frac{\cancel{x^2}\left(\frac{2}{x} - 3\right)}{x^3\left(1 + \frac{1}{x}\right)} = \frac{\frac{2}{x} - 3}{x\left(1 + \frac{1}{x}\right)}$$

$$\lim_{x \to -\infty} \frac{2}{x} - 3 = -3$$

$$\lim_{x \to -\infty} x\left(1 + \frac{1}{x}\right) = -\infty \cdot 1 = -\infty$$

Per il teorema sul limite del quoziente:

$$\lim_{x \to -\infty} \frac{\frac{2}{x} - 3}{x\left(1 + \frac{1}{x}\right)} = \frac{-3}{-\infty} = 0$$

⑧ $\lim_{x \to +\infty} \left(\sqrt{2x+1} - \sqrt{x}\right)$

Il limite ha senso in quanto il dominio della funzione $f(x)$ è $[0, +\infty[$ e quindi $+\infty$ è un punto di accumulazione per dom$f$.

$$\lim_{x \to +\infty} \sqrt{x\left(2+\frac{1}{x}\right)} - \sqrt{x} = \sqrt{x} \cdot \left(\underbrace{\sqrt{2+\frac{1}{x}}}_{\downarrow \sqrt{2}} - 1\right)$$

Siccome $(\sqrt{2}-1) > 0$, per il teorema sul limite del prodotto:

$$\lim_{x \to +\infty} \sqrt{x}\left(\sqrt{2+1/x} - 1\right) = +\infty \cdot \left(\sqrt{2} - 1\right) = +\infty$$

LIMITI NOTEVOLI

LIMITE NOTEVOLE IN FORMA SINTETICA

$\frac{\sin(x)}{x} \to 1$ se $x \to 0$

$\frac{\ln(1+x)}{x} \to 1$ se $x \to 0$

$\frac{\log_a(1+x)}{x} \to \frac{1}{\ln(a)}$ se $x \to 0$

$\frac{e^x-1}{x} \to 1$ se $x \to 0$

$\frac{a^x-1}{x} \to \ln(a)$ se $x \to 0$

$\left(1+\frac{1}{x}\right)^x \to e$ se $x \to \pm\infty$

$\frac{(1+x)^c-1}{x} \to c$ se $x \to 0$
per ogni $c \in \mathbb{R}$

$\frac{1-\cos(x)}{x^2} \to \frac{1}{2}$ se $x \to 0$

$\frac{\tan(x)}{x} \to 1$ se $x \to 0$

$\frac{\arcsin(x)}{x} \to 1$ se $x \to 0$

$\frac{\arctan(x)}{x} \to 1$ se $x \to 0$

LIMITE NOTEVOLE IN FORMA GENERALE

$\frac{\sin(f(x))}{f(x)} \to 1$ se $f(x) \to 0$

$\frac{\ln(1+f(x))}{f(x)} \to 1$ se $f(x) \to 0$

$\frac{\log_a(1+f(x))}{f(x)} \to \frac{1}{\ln(a)}$ se $f(x) \to 0$

$\frac{e^{f(x)}-1}{f(x)} \to 1$ se $f(x) \to 0$

$\frac{a^{f(x)}-1}{f(x)} \to \ln(a)$ se $f(x) \to 0$

$\left(1+\frac{1}{f(x)}\right)^{f(x)} \to e$ se $f(x) \to \pm\infty$

$\frac{(1+f(x))^c-1}{f(x)} \to c$ se $f(x) \to 0$
per ogni $c \in \mathbb{R}$

$\frac{1-\cos(f(x))}{(f(x))^2} \to \frac{1}{2}$ se $f(x) \to 0$

$\frac{\tan(f(x))}{f(x)} \to 1$ se $f(x) \to 0$

$\frac{\arcsin(f(x))}{f(x)} \to 1$ se $f(x) \to 0$

$\frac{\arctan(f(x))}{f(x)} \to 1$ se $f(x) \to 0$

ESEMPI

① $$\lim_{x\to 0^+} \frac{e^x - 2^x}{x^2+2x} = \left[\frac{0}{0}\right]$$ Procedendo per sostituzione avremmo una forma indeterminata

(•) $$\lim_{x\to 0} \frac{e^x-1}{x} = 1$$ (••) $$\lim_{x\to 0} \frac{a^x-1}{x} = \ln(a)$$

da qui le equivalenze asintotiche

(•) $e^x-1 \sim x$ per $x\to 0$ (••) $a^x-1 \sim x\ln(a)$ per $x\to 0$

$$\lim_{x\to 0^+} \frac{e^x-1+1-2^x}{x^2+2x} = \lim_{x\to 0^+} \left[\frac{e^x-1}{x^2+2x} + \frac{1-2^x}{x^2+2x}\right]$$

con (•) e (••):

$$\lim_{x\to 0^+} \left[\frac{x}{x^2+2x} - \frac{\ln(2)x}{x^2+2x}\right] =$$

$$= \lim_{x\to 0^+} \left[\frac{x}{x(x+2)} - \frac{\ln(2)x}{x(x+2)}\right] =$$

$$= \lim_{x\to 0^+} \left[\frac{1}{x+2} - \frac{\ln(2)}{x+2}\right] = \frac{1-\ln(2)}{2}$$

② $$\lim_{x\to 0} \frac{\operatorname{sen}(2x)}{x}$$

Moltiplichiamo e dividiamo per 2

$$\lim_{x\to 0} \frac{\operatorname{sen}(2x)}{x} = \lim_{x\to 0} \frac{2}{2} \cdot \frac{\operatorname{sen}(2x)}{x} =$$

$$= \lim_{x\to 0} 2 \cdot \frac{\operatorname{sen}(2x)}{2x}$$

a questo punto, ponendo $y = 2x$, dato che:

$$\lim_{y\to 0} \frac{\operatorname{sen} y}{y} = 1$$

otteniamo:

$$\lim_{x\to 0} 2 \cdot \frac{\operatorname{sen}(2x)}{2x} = \lim_{y\to 0} 2 \cdot \frac{\operatorname{sen} y}{y} = 2 \cdot 1 = 2$$

(3) $\lim_{x \to 0} \dfrac{1-\cos x}{\text{sen}(4x)}$

Riscriviamo il limite in questo modo:

$$\lim_{x \to 0} \frac{1-\cos x}{\text{sen}(4x)} = \lim_{x \to 0} (1-\cos x) \cdot \frac{1}{\text{sen}(4x)}$$

$$= \lim_{x \to 0} \frac{x^2}{x^2} \cdot (1-\cos x) \cdot \frac{4x}{4x} \cdot \frac{1}{\text{sen}(4x)} =$$

$$= \lim_{x \to 0} x^{\cancel{2}} \cdot \frac{1-\cos x}{x^2} \cdot \frac{1}{4\cancel{x}} \cdot \frac{4x}{\text{sen}(4x)}$$

$$= \lim_{x \to 0} \frac{x}{4} \cdot \frac{1-\cos x}{x^2} \cdot \frac{4x}{\text{sen}(4x)}$$

Per i limiti notevoli:

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2} \quad ; \quad \lim_{x \to 0} \frac{4x}{\text{sen}(4x)} = 1$$

$$\Rightarrow \lim_{x \to 0} \frac{x}{4} \cdot \frac{1-\cos x}{x^2} \cdot \frac{4x}{\text{sen}(4x)} = 0 \cdot \frac{1}{2} \cdot 1 = 0$$

(4) $\lim_{x \to 0} \dfrac{1-\cos^3 x}{3x^2}$ N.B.: Il numeratore è una differenza di cubi

REGOLA DEL FALSO QUADRATO
$a^3 - b^3 = (a-b)(a^2+b^2+ab)$

$$\lim_{x \to 0} \frac{1-\cos^3 x}{3x^2} = \lim_{x \to 0} \frac{(1-\cos x)(1+\cos x+\cos^2 x)}{3x^2} =$$

$$= \lim_{x \to 0} \frac{1-\cos x}{3x^2} \cdot (1+\cos x+\cos^2 x) =$$

$$= \lim_{x \to 0} \frac{1}{3} \cdot \frac{1-\cos x}{x^2} \cdot (1+\cos x+\cos^2 x)$$

Poiché risulta:

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2} \quad ; \quad \lim_{x \to 0} (1+\cos x+\cos^2 x) = 1+1+1^2 = 3$$

Abbiamo:

$$\lim_{x \to 0} \frac{1}{3} \cdot \frac{1-\cos x}{x^2} \cdot (1+\cos x+\cos^2 x) = \frac{1}{3} \cdot \frac{1}{2} \cdot 3 = \frac{1}{2}$$

⑤ $$\lim_{x\to 0^-} \frac{\sqrt{1-\cos(5x)}}{x}$$

Portiamo la x dentro la radice, facendo attenzione al fatto che, trattandosi di un limite per $x\to 0^-$, la x è negativa:

$$\lim_{x\to 0^-} \frac{\sqrt{1-\cos(5x)}}{x} = \lim_{x\to 0^-} -\sqrt{\frac{1-\cos(5x)}{x^2}}$$

Moltiplichiamo e dividiamo dentro la radice per 25:

$$\lim_{x\to 0^-} -\sqrt{\frac{25}{25}\cdot\frac{1-\cos(5x)}{x^2}} =$$

$$\lim_{x\to 0^-} -\sqrt{25\cdot\frac{1-\cos(5x)}{25x^2}} =$$

$$\lim_{x\to 0^-} -\sqrt{25\cdot\underbrace{\frac{1-\cos(5x)}{(5x)^2}}_{\text{limite notevole}}} = -\sqrt{25\cdot\frac{1}{2}} = -\frac{5}{\sqrt{2}} = -\frac{5\sqrt{2}}{2}$$

⑥ $$\lim_{x\to 1} \frac{1-e^{(1-x)^2}}{3(x-1)^2}$$

Mettendo un $(-)$ in evidenza e, poiché $(1-x)^2=(x-1)^2$, abbiamo:

$$\lim_{x\to 1} \frac{1-e^{(1-x)^2}}{3\cdot(x-1)^2} = \lim_{x\to 1} -\frac{e^{(x-1)^2}-1}{3\cdot(x-1)^2} = \lim_{x\to 1} -\frac{1}{3}\cdot\frac{e^{(x-1)^2}-1}{(x-1)^2}$$

Ponendo ora $y = x-1$:

$$\lim_{x\to 1} -\frac{1}{3}\cdot\frac{e^{(x-1)^2}-1}{(x-1)^2} = \lim_{y\to 0} -\frac{1}{3}\cdot\frac{e^{y^2}-1}{y^2}$$

Ponendo ora $z = y^2$:

$$\lim_{y\to 0} -\frac{1}{3}\cdot\frac{e^{y^2}-1}{y^2} = \lim_{z\to 0} -\frac{1}{3}\cdot\frac{e^{z}-1}{z} = -\frac{1}{3}\cdot 1 = -\frac{1}{3}$$

(7) $$\lim_{x\to 0} \frac{x^2 \cdot \ln(1+2x)}{(2\cdot\cos(3x)-2)\cdot \operatorname{sen}x}$$

Riscriviamo il limite nel modo seguente:

$$\lim_{x\to 0} -\frac{x^2}{2}\cdot\frac{1}{1-\cos(3x)}\cdot\ln(1+2x)\cdot\frac{1}{\operatorname{sen}x} =$$

$$= \lim_{x\to 0} -\frac{x^2}{2}\cdot\frac{9x^2}{9x^2}\cdot\frac{1}{1-\cos(3x)}\cdot\frac{2x}{2x}\cdot\ln(1+2x)\cdot\frac{x}{x}\cdot\frac{1}{\operatorname{sen}x} =$$

Riportiamo il limite ai limiti notevoli:

$$= \lim_{x\to 0} -\frac{x^2}{2}\cdot\frac{1}{9x^2}\cdot\frac{9x^2}{1-\cos(3x)}\cdot 2x\cdot\frac{\ln(1+2x)}{2x}\cdot\frac{1}{x}\cdot\frac{x}{\operatorname{sen}x} =$$

$$= \lim_{x\to 0} -\frac{\cancel{x^2}}{\cancel{2}}\cdot\frac{1}{9\cancel{x^2}}\cdot\cancel{2x}\cdot\frac{1}{\cancel{x}}\cdot\frac{(3x)^2}{1-\cos(3x)}\cdot\frac{\ln(1+2x)}{2x}\cdot\frac{x}{\operatorname{sen}x} =$$

$$= \lim_{x\to 0} -\frac{1}{9}\cdot 2\cdot 1\cdot 1 = -\frac{2}{9}$$

(8) $$\lim_{x\to 0} (e^x-1)\cdot\frac{\operatorname{tg}x-\operatorname{sen}x}{x^4}$$

$$\lim_{x\to 0} \frac{e^x-1}{x}\cdot\cancel{x}\cdot\frac{\frac{\operatorname{sen}x}{\cos x}-\operatorname{sen}x}{x^{\cancel{4}\,3}} = \frac{e^x-1}{x}\cdot\frac{\operatorname{sen}x\left(\frac{1}{\cos x}-1\right)}{x^3} =$$

$$\lim_{x\to 0} \frac{e^x-1}{x}\cdot\frac{\operatorname{sen}x}{x}\cdot\frac{\frac{1-\cos x}{\cos x}}{x^2} =$$

$$\lim_{x\to 0} \frac{e^x-1}{x}\cdot\frac{\operatorname{sen}x}{x}\cdot\frac{1}{x^2}\cdot\frac{(1-\cos x)}{\cos x} =$$

$$\lim_{x\to 0} \frac{e^x-1}{x}\cdot\frac{\operatorname{sen}x}{x}\cdot\frac{1-\cos x}{x^2}\cdot\frac{1}{\cos x} =$$

$$\lim_{x\to 0} 1\cdot 1\cdot\frac{1}{2}\cdot 1 = \frac{1}{2}$$

INCONTRO V

Richiami sui limiti complessi

(1)

$$\lim_{x \to 1} \frac{x^3+x-2}{x^3-x^2-x+1}$$

Non è possibile procedere per sostituzione: si otterrebbe una forma indeterminata $[0/0]$

$$\frac{x^3+x-2}{x^2(x-1)-(x-1)} \leadsto \text{scompongo il det.}$$

$$\lim_{x \to 1} \frac{x^3+x-2}{(x^2-1)(x-1)} = \lim_{x \to 1} \frac{x^3+x-2}{(x+1)\cdot(x-1)^2} =$$

$$\lim_{x \to 1} \frac{(x^2+x+2)(x-1)}{(x+1)\cdot(x-1)^2}$$

Ruffini:

	1	0	1	-2
1		1	1	2
	1	1	2	//

N.B.: Il limite cambia se mi avvicino da 1^- a 1^+ quindi li calcolo separatamente

$$\lim_{x \to 1^-} \frac{x^2+x+2}{(x+1)(x-1)} = \lim_{x \to 1^-} \frac{4}{2\cdot 0^-} = -\infty$$

$$\lim_{x \to 1^+} \frac{x^2+x+2}{(x+1)(x-1)} = \lim_{x \to 1^+} \frac{4}{2\cdot 0^+} = +\infty$$

= Il limite non esiste

(2) $$\lim_{x \to 0} \frac{\sqrt{1+x^2}-1}{x^2}$$

Ricorda formula falso quadrato

$$\frac{\left(\sqrt[3]{1+x^2}-1\right)\cdot\left(\sqrt[3]{(1+x^2)^2}+1+\sqrt[3]{1+x^2}\right)}{x^2\cdot\left(\sqrt[3]{(1+x^2)^2}+1+\sqrt[3]{1+x^2}\right)}$$

Moltiplico e divido per la stessa quantità

$$\lim_{x \to 0} \frac{\cancel{1}+x^2-\cancel{1}}{x^2\cdot\left(\sqrt[3]{(1+x^2)^2}+1+\sqrt[3]{1+x^2}\right)} =$$

$$\lim_{x \to 0} \frac{1}{1+1+1} = \frac{1}{3}.$$

③ $\lim_{x\to 0} \dfrac{(e^{ax}-1)\cdot \log \cos x}{tg^3\left(\frac{x}{a}\right)}$

$\rightarrow \cos x = \sqrt{1-sen^2 x}$

$\rightarrow \log \sqrt[n]{b} = \frac{1}{n}\cdot \log b$

$$\lim_{x\to 0} \frac{\frac{e^{ax}-1}{ax}\cdot ax \cdot \log\sqrt{1-sen^2 x}}{\left(\frac{tg\, x/a}{\frac{x}{a}}\right)^3 \cdot \frac{x^3}{a^3}} =$$

$$\lim_{x\to 0} \frac{\frac{e^{ax}-1}{ax}\cdot ax\cdot \frac{1}{2}\cdot \frac{\log(1+(-sen^2 x))}{-sen^2 x}\cdot(-sen^2 x)}{x^3/a^3} =$$

$$\lim_{x\to 0} \frac{\underbrace{\frac{e^{ax}-1}{ax}}_{\to 1}\cdot a^4\cdot\frac{1}{2}\cdot \underbrace{\frac{\log[1+(-sen^2 x)]}{-sen^2 x}}_{\to 1}\cdot \underbrace{\left(-\frac{sen^2 x}{x^2}\right)}_{\to -1}}{\underbrace{\left(\frac{tg\, x/a}{x/a}\right)^3}_{\to 1}} = -\frac{a^4}{2}$$

Limiti di successioni

④ $\lim_{n\to+\infty} \dfrac{(n+1)^6-(n-1)^6}{(n+1)^5} =$

$$\frac{[(n+1)^3-(n-1)^3][(n+1)^3+(n-1)^3]}{(n+1)^5}$$ ho scomposto il num.

$$\lim_{n\to+\infty} \frac{[(n+1)-(n-1)][(n+1)^2+(n-1)^2+(n+1)(n-1)]}{(n+1)^5}\cdot$$

$$\cdot\ [(n+1)+(n-1)]\cdot[(n+1)^2+(n-1)^2-(n+1)(n-1)] =$$

$$\lim_{n\to+\infty} \frac{2\cdot(3n^2+1)\cdot(2n)\cdot(n^2+3)}{(n+1)^5} =$$

$$\lim_{n\to+\infty} \frac{4\cancel{n}\cdot \cancel{n^2}\left(3+\underbrace{1/n^2}_{\to 0}\right)\cdot\cancel{n^2}\left(1+\underbrace{3/n^2}_{\to 0}\right)}{(n+1)^5 = \cancel{n^5}\left(1+\underbrace{1/n}_{\to 0}\right)^5} = \lim_{n\to+\infty} \frac{4\cdot 3\cdot 1}{1} = 12.$$

5) $$\lim_{x\to+\infty} \left(\sqrt{n+1}-\sqrt{n}\right)\cdot\sqrt{n} =$$

$$\lim_{x\to+\infty} \frac{\left(\sqrt{n+1}-\sqrt{n}\right)}{\sqrt{n+1}+\sqrt{n}} \cdot\sqrt{n} \cdot \left(\sqrt{n+1}+\sqrt{n}\right) =$$

$$\lim_{x\to+\infty} \frac{\cancel{n}+1-\cancel{n}}{\sqrt{n+1}+\sqrt{n}}\cdot\sqrt{n} =$$

$$\lim_{x\to+\infty} \frac{1}{\frac{\sqrt{n+1}}{\sqrt{n}}+\cancel{\sqrt{n}}/\cancel{\sqrt{n}}} = \frac{1}{\sqrt{\frac{n+1}{n}}+1}$$

$$\lim_{x\to+\infty} \frac{1}{\sqrt{1+\underbrace{1/n}_{\to 0}}+1} = \lim_{x\to+\infty} \frac{1}{1+1} = \frac{1}{2}$$

6) $$\lim_{n\to+\infty} \frac{3^n - 3^{n\cdot\log n}}{n^n} =$$

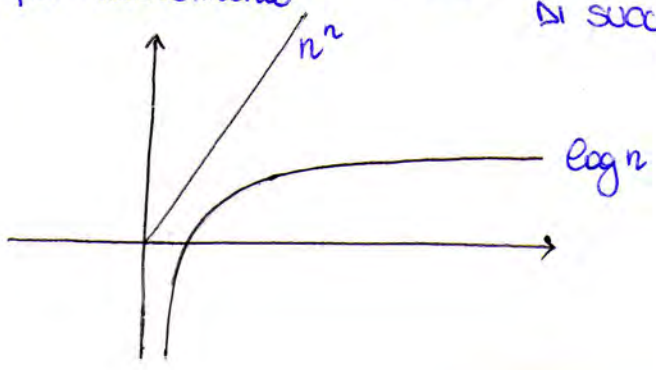
$$\lim_{n\to+\infty} \frac{3^n}{n^n} - \frac{3^{\log n^n}}{n^n} + \frac{1}{n^n} - \frac{1}{n^n} =$$

$$\lim_{n\to+\infty} \frac{3^n}{n^n} - \left(\frac{3^{\log n^n}-1}{n^n}\right) - \frac{1}{n^n} =$$

$$\lim_{n\to+\infty} \underbrace{\frac{3^n}{n^n}}_{\to 0} - \underbrace{\frac{3^{\log n^n}-1}{\log n^n}}_{\to \log 3} \cdot \underbrace{\frac{\log n^n}{n^n}}_{\to 0} - \underbrace{\frac{1}{n^n}}_{\to 0}$$

perché n^n tende a $+\infty$ più velocemente

LIMITE NOTEVOLE DI SUCCESSIONI



METODO DEL RAPPORTO

$$\lim_{n \to +\infty} a_n$$

$$\lim_{n \to +\infty} \frac{a_{n+1}}{a_n} \begin{cases} < 1 & \text{succ converge} \\ = 1 & \\ > 1 & \text{" diverge a } +\infty \end{cases}$$

(7) $\lim_{n \to +\infty} \frac{2^n}{n}$ Quindi $a_n = \frac{2^n}{n}$

Applichiamo il metodo del rapporto:

$$\lim_{n \to +\infty} \frac{a_{n+1}}{a_n} \Rightarrow \lim_{n \to +\infty} \qquad a_n = \frac{2^n}{n} \qquad a_{n+1} = \frac{2^{n+1}}{n+1}$$

$$\frac{a_{n+1}}{a_n} = \frac{2^{n+1}}{n+1} \cdot \frac{n}{2^n}$$

$$\Rightarrow \lim_{n \to +\infty} \frac{2^{\cancel{n}+1-\cancel{n}}}{\frac{n+1}{n}} = \frac{2}{\frac{n+1}{n}} \Rightarrow \lim_{n \to +\infty} \frac{2}{1 + \underbrace{\frac{1}{n}}_{\to 0}}$$

$\lim_{n \to +\infty} \frac{2}{1} = 2$ Poiché > 1,

il limite diverge a $+\infty$

METODO DELLA RADICE

$$\lim_{n \to +\infty} \sqrt[n]{2^n + n}$$

$a_n = 2^n + n$

(8) $a_{n+1} = 2^{n+1} + n + 1$

$$\frac{a_{n+1}}{a_n} = \frac{2^{n+1} + n + 1}{2^n + n}$$

$$\lim_{n \to +\infty} \frac{a_{n+1}}{a_n} = \frac{\cancel{2^n}\left(2 + \underbrace{\frac{n}{2^n}}_{\to 0} + \underbrace{\frac{1}{2^n}}_{\to 0}\right)}{\cancel{2^n}\left(1 + \underbrace{\frac{n}{2^n}}_{\to 0}\right)} = 2$$

⑨ $\lim_{n \to +\infty} \sqrt[n]{n^2 - 2\cos n}$

$a_n = n^2 - 2\cos n$

$a_{n+1} = (n+1)^2 - 2\cos(n+1)$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2 - 2\cos(n+1)}{n^2 - 2\cos n}$$

$$\lim_{n \to +\infty} \frac{\cancel{n^2} \cdot \left[\left(\frac{n+1}{n} \right)^2 - 2 \cdot \frac{\cos(n+1)}{n^2} \right]}{\cancel{n^2} \cdot \left(1 - 2 \frac{\cos n}{n^2} \right)}$$

$$\lim_{n \to +\infty} \frac{\left(1 + \underbrace{\frac{1}{n}}_{\to 0}\right)^2 - 2 \cdot \frac{\cos(n+1)}{n^2} \to 0}{1 - 2 \cdot \frac{\cos n}{n^2} \to 0} =$$

num. finito

den. indefinito

$$\lim_{n \to +\infty} \frac{1 - 2 \cdot 0}{1 - 2 \cdot 0} = 1$$

INCONTRO VI

Es. limiti misti

(1)

$$\lim_{n \to +\infty} \frac{n^{n+1} + 3(n+1)^{n+1}}{n^n + n!} \cdot \operatorname{sen}\frac{\pi}{n} =$$

$$\lim_{n \to +\infty} \frac{\cancel{n^n} \cdot \left[n + 3 \cdot \frac{(n+1)^n}{n^n} \cdot (n+1)\right]}{\cancel{n^n} \cdot \left(1 + \frac{n!}{n^n}\right)} \cdot \operatorname{sen}\frac{\pi}{n} =$$

$$\lim_{n \to +\infty} \frac{n + 3 \cdot \left(\frac{n+1}{n}\right)^n \cdot (n+1)}{1 + \frac{n!}{n^n}} \cdot \operatorname{sen}\frac{\pi}{n} =$$

$$\lim_{n \to +\infty} \frac{n + 3 \cdot \left(1 + \frac{1}{n}\right) \cdot (n+1)}{1 + \frac{n!}{n^n}} \cdot \operatorname{sen}\frac{\pi}{n}$$

LIMITE NOTEVOLE DI SUCCESSIONI: $\lim \frac{\operatorname{sen} x}{x} \to 1$ $\quad \lim_{n \to +\infty} \frac{\operatorname{sen} 1/n}{1/n}$

$$\lim_{n \to +\infty} \frac{\cancel{n} \cdot \left[1 + 3 \cdot \overset{e}{\left(1 + \frac{1}{n}\right)^n} \cdot \overset{1}{\left(1 + \frac{1}{n}\right)}\right]}{1 + \underset{0}{\frac{n!}{n^n}}} \cdot \overset{1}{\frac{\operatorname{sen} \pi/n}{\pi/n}} \cdot \frac{\pi}{\cancel{n}} =$$

$$\lim_{n \to +\infty} \frac{1 + 3 \cdot e \cdot 1}{1} \cdot \pi = (1 + 3e) \cdot \pi$$

LIMITI NOTEVOLI DI SUCCESSIONI

(2)

$$\lim_{n\to+\infty} \frac{\operatorname{sen}^2 \cdot \frac{3}{n} - \frac{1}{n^2}}{\cos\frac{2}{n} - 1}$$

$$\lim_{n\to+\infty} \frac{\frac{1/n^2 \cdot \left(\frac{\operatorname{sen}^2 3/n}{1/n^2} - 1\right)}{}}{\frac{\cos\frac{2}{n} - 1}{1/n^2}} =$$

$$\lim_{n\to+\infty} \frac{\left[\frac{\operatorname{sen}\left(\frac{3}{n}\right)}{\frac{3}{n}}\right]^2 \cdot 9 - 1 \;\; \Big\} = 8}{-\underbrace{\frac{-\cos\frac{2}{n} + 1}{4/n^2}}_{\to 1/2} \cdot 4 \;\; \Big\} -\frac{1}{\cancel{2}} \cdot \cancel{4}\,2} = -4$$

(3)

$$\lim_{n\to+\infty} \frac{\log\left(1 + \frac{1}{n^3}\right)}{\operatorname{sen}\frac{1}{n^2} \cdot \left(1 - e^{2/n}\right)}$$

$$\lim_{n\to+\infty} \frac{\underbrace{\frac{\log\left(1 + \frac{1}{n^3}\right)}{1/n^3}}_{\to 1} \cdot \frac{1}{\cancel{n^3}}}{-\underbrace{\frac{\operatorname{sen}\frac{1}{n^2}}{1/n^2}}_{\to 1} \cdot \frac{1}{\cancel{n^2}} \cdot \underbrace{\frac{e^{2/n} - 1}{\frac{2}{n}}}_{\to 1} \cdot \frac{1}{\cancel{n}} \cdot 2} = \frac{1}{-1 \cdot 1 \cdot 2} = -\frac{1}{2}$$

④ $$\lim_{n\to+\infty} \frac{n^{n+1}+2n!}{(n-1)^n+2n^n}\cdot \tan\frac{2\pi}{n}$$

metto in evidenza il teremine che tende + veloce:

$$\lim_{n\to+\infty} \frac{n^{\cancel{n}+1}}{n^{\cancel{n}}}\cdot \frac{1+2\,\frac{n!}{n^{n+1}}}{\left(\frac{n-1}{n}\right)^n+2}\cdot \tan\frac{2\pi}{n} =$$

n^{n+1} più veloce di $2n!$

$$\lim_{n\to+\infty} n\cdot \frac{1+2\,\frac{n!}{n^{n+1}}}{\left[\left(1+\frac{1}{-n}\right)^{-n}\right]^{-1}+2}\cdot \tan\frac{2\pi}{n} =$$

$$\lim_{n\to+\infty} \frac{1+\underbrace{2\,\frac{n!}{n^{n+1}}}_{\to 0}}{\Big[\underbrace{\left(1+\frac{1}{-n}\right)^{-n}}_{\downarrow\, e}\Big]^{-1}+2}\cdot \underbrace{\frac{\tan 2\pi/n}{\frac{2\pi}{n}}}_{\downarrow\, 1}\cdot 2\pi =$$

$$\lim_{n\to+\infty} \frac{1+0}{e^{-1}+2}\cdot 2\pi = \frac{2\pi}{e^{-1}+2} = \frac{2\pi\cdot e}{1+2e}$$

⑤ $$\lim_{n\to+\infty} \frac{n!\cdot(n+2)^n}{n^n\cdot(n+1)!}\cdot \ln 3^n =$$

RICORDA:
$(n+1)! = (n+1)\cdot(n)\cdot(n-1)\cdot(n-2)\ldots$

$$\lim_{n\to+\infty} \frac{\cancel{n!}}{(n+1)\cdot\cancel{n!}}\cdot\left(\frac{n+2}{n}\right)^n\cdot n\cdot\ln 3 =$$

$$\lim_{n\to+\infty} \underbrace{\frac{n}{n+1}}_{\downarrow}\cdot\Big[\underbrace{\left(1+\frac{1}{\frac{n}{2}}\right)^{n/2}}_{\downarrow\, e}\Big]^2\cdot\ln 3 = 1\cdot e^2\cdot\ln 3 =$$

$$\frac{\cancel{n}}{\cancel{n}(1+1/n)} = 1$$

$$= e^2\cdot\ln 3$$

⑥

$$\lim_{n\to+\infty} \sqrt[n]{2^n \cdot \frac{(n+1)!}{n^n+\log n} \cdot \operatorname{tg}\frac{1}{n+1}}$$

$\boxed{\text{RAPPORTO:}}$

$$\lim_{n\to+\infty} \frac{a_{n+1}}{a_n} = l$$

$$a_{n+1} = 2^{n+1} \cdot \frac{(n+2)!}{(n+1)^{n+1}+\log(n+1)} \cdot \operatorname{tg}\frac{1}{n+2}$$

$$\frac{a_{n+1}}{a_n} = \frac{2^{n+1}}{2^n} \cdot \frac{(n+2)!}{(n+1)^{n+1}+\log(n+1)} \cdot \frac{n^n+\log n}{(n+1)!} \cdot \frac{\operatorname{tg}(1/n+2)}{\operatorname{tg}(1/n+1)} =$$

$$\lim_{n\to+\infty} 2 \cdot \frac{(n+2)\cdot\cancel{(n+1)!}}{(n+1)^n\cdot(n+1)+\log(n+1)} \cdot \frac{n^n+\log n}{\cancel{(n+1)!}} \cdot \frac{\operatorname{tg}(1/n+2)}{\operatorname{tg}(1/n+1)} =$$

$$\lim_{n\to+\infty} 2 \cdot \frac{n^n\left(1+\frac{\log n}{n^n}\right)}{(n+1)^n + \frac{\log(n+1)}{n+1}} \cdot \frac{\frac{\operatorname{tg} 1/n+2}{1/n+2}}{\frac{\operatorname{tg} 1/n+1}{1/n+1}} \cdot \frac{\frac{1}{(n+\cancel{2})}(n+\cancel{2})}{\frac{1}{(n+\cancel{1})}(n+\cancel{1})} =$$

$$\lim_{n\to+\infty} 2 \cdot \frac{n^n\cdot\left(1+\overbrace{\frac{\log n}{n^n}}^{\to 0}\right)}{(n+1)^n\cdot\left(1+\underbrace{\frac{\log(n+1)}{(n+1)^{n+1}}}_{\to 0}\right)} \cdot \frac{\overbrace{\frac{\operatorname{tg} 1/(n+2)}{1/(n+2)}}^{\to 1}}{\underbrace{\operatorname{tg} 1/(n+1)/1/(n+1)}_{\to 1}} =$$

n^n tende a $+\infty$ + velocem. di $\log n$.

$$\lim_{n\to+\infty} 2 \cdot \frac{\cancel{n^n}\cdot 1}{\cancel{n^n}\cdot\frac{(n+1)^n}{n^n}\cdot(1)} \cdot 1 = \lim_{n\to+\infty} 2 \cdot \frac{1}{\underbrace{\left(\frac{n+1}{n}\right)^n}_{e}} = \frac{2}{e}$$

LIMITI PER SOSTITUZIONE

(7)
$$\lim_{x\to\pi} \frac{1+\cos x}{(\pi-x)^2}$$

$t=\pi-x$

la nuova variabile deve tendere al valore del limite notevole:
$$\begin{cases} x\to\pi \\ t\to 0 \end{cases}$$

$$= \lim_{t\to 0} \frac{1+\cos(\pi-t)}{t^2} =$$

$$= \lim_{t\to 0} \frac{1-\cos t}{t^2} = \frac{1}{2}$$

(8)
$$\lim_{x\to+\infty} x\cdot\left(1-e^{x/1+x^2}\right)$$

$$\lim_{x\to+\infty} -x\cdot\left(e^{\frac{x}{1+x^2}}-1\right) = -x\cdot\frac{e^{\frac{1}{x+1/x}}-1}{\frac{1}{x+1/x}}\cdot\frac{1}{x+1/x} =$$

$$\lim_{x\to+\infty} \frac{e^{\frac{1}{x+1/x}}-1}{\frac{1}{x+1/x}}\cdot\left(-\frac{x}{x+\frac{1}{x}}\right) =$$

$$\lim_{x\to+\infty} \frac{e^{\frac{1}{x+1/x}}-1}{\frac{1}{x+\frac{1}{x}}}\cdot\left(-\frac{1}{1+\left(\frac{1}{x^2}\right)}\right) =$$

$\frac{1}{x+1/x} = t \Rightarrow t\to 0$

$\frac{1}{x^2}\to 0$

$$\lim_{t\to 0}\frac{e^t-1}{t}\cdot\lim_{x\to+\infty}\left(-\frac{1}{1+1/x^2}\right) = 1\cdot(-1) = -1$$

$\to 1$, $\to 0$

(9)
$$\lim_{x\to\pi/4} \frac{\mathrm{tg}\,2x}{\mathrm{tg}\left(\frac{\pi}{4}+x\right)}$$

RICORDA:
$$\mathrm{tg}\,2x = \frac{2\,\mathrm{tg}x}{1-\mathrm{tg}^2x} \qquad \mathrm{tg}(\alpha+x) = \frac{\mathrm{tg}\alpha+\mathrm{tg}x}{1-\mathrm{tg}\alpha\cdot\mathrm{tg}x}$$

$$\lim_{x\to\pi/4}\frac{\frac{2\,\mathrm{tg}x}{(1+\mathrm{tg}x)(1-\mathrm{tg}x)}}{\frac{1+\mathrm{tg}x}{1-\mathrm{tg}x}} = \frac{2\,\mathrm{tg}x}{(1-\cancel{\mathrm{tg}x})(1+\mathrm{tg}x)}\cdot\frac{1-\cancel{\mathrm{tg}x}}{1+\mathrm{tg}x} = \frac{2\,\mathrm{tg}x}{(1+\mathrm{tg}x)^2} = \frac{2}{4} = \frac{1}{2}$$